**Magri, Giorgio**

# IDEMPOTENCE AND THE TRIANGULAR INEQUALITY: SOME CONSEQUENCES OF MCCARTHY'S CATEGORICITY GENERALIZATION

■ A phonological grammar $G$ is called *idempotent* provided it realizes faithfully any phonotactically licit form. This condition is formalized through the implication (1). The antecedent says that $G$ maps some UR $\mathbf{a}$ to some candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ whose UR is indeed $\mathbf{a}$ and whose SR is some form $\mathbf{b}$ related to $\mathbf{a}$ through a correspondence relation (CR) $\rho_{\mathbf{a},\mathbf{b}}$ (McCarthy and Prince 1995). The consequent of (1) says that $G$ then maps that form $\mathbf{b}$ (construed as an UR) into the *identity candidate* $(\mathbf{b}, \mathbf{b}, \mathbb{I}_{\mathbf{b},\mathbf{b}})$ whose UR and SR are both $\mathbf{b}$ and whose CR is the identity $\mathbb{I}_{\mathbf{b},\mathbf{b}}$ (which sets each segment of the string $\mathbf{b}$ in correspondence with itself). Idempotence is crucial in the literature on the early acquisition of phonotactics as it ensures the learner can safely posit a faithful UR for any licit training SR.

(1) **If:** $G(\mathbf{a}) = (\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$
**Then:** $G(\mathbf{b}) = (\mathbf{b}, \mathbf{b}, \mathbb{I}_{\mathbf{b},\mathbf{b}})$

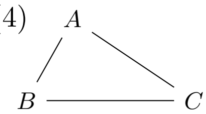
■ Which conditions on the constraints ensure idempotence? Magri (2015) shows that idempotence holds of each grammar in an OT typology provided each faithfulness constraint $F$ in the corresponding constraint set satisfies the *OT faithfulness idempotence implication* (FII$^{\mathrm{OT}}$) in (2) for any candidates $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ and $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$. The antecedent of the implication says that the candidate $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$ does not violate the constraint $F$. The consequent says that the candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ violates $F$ at least as much as the candidate $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}})$ which pairs the UR $\mathbf{a}$ with the SR $\mathbf{c}$ through the CR $\rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}}$ which is the *composition* of the two CRs $\rho_{\mathbf{a},\mathbf{b}}$ and $\rho_{\mathbf{b},\mathbf{c}}$.

(2) **If:** $F\big(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}}\big) = 0$
**Then:** $F\big(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}}\big) \leq F\big(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}}\big)$

■ The **first contribution** of this paper is an extension of the theory of idempotence from OT to HG: I show that idempotence holds of each grammar in an HG typology provided each faithfulness constraint $F$ in the corresponding constraint set satisfies the *HG faithfulness idempotence implication* (FII$^{\mathrm{HG}}$) in (3) for any candidates $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ and $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$ and any threshold $\xi \geq 0$ (if $\xi < 0$, the antecedent is trivially false because constraints are non-negative). Obviously, (3) is stronger than (2) in the general case, as the latter corresponds to the former with $\xi = 0$. This makes sense: as HG typologies are generally larger than OT ones, a stronger condition is needed to discipline all HG grammars to abide by idempotence.

(3) **If:** $F\big(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}}\big) \leq \xi$
**Then:** $F\big(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}}\big) \leq F\big(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}}\big) + \xi$

■ Do these rather technical OT/HG idempotence conditions (2)-(3) admit any intuitive interpretation? To address this question, I probe deeper into the formal underpinning of the theory of faithfulness. Intuitively, faithfulness constraints measure the distance between URs and SRs along various phonologically relevant dimensions. It thus makes sense to investigate whether faithfulness constraints satisfy the axioms of the abstract theory of distance. A distance *dist* pairs two points $A$ and $B$ with a non-negative number $dist(A, B)$ in compliance with various axioms. One of these axioms requires that $dist(A, C) \leq dist(A, B) + dist(B, C)$. This axiom is known as the *triangular inequality* as it can be reinterpreted as saying that a side $AC$ of a triangle is never longer than the sum of the other two sides $AB$ and $BC$ in (4). These considerations motivate the *faithfulness triangular inequality* (FTI) in (5). It says that the sum of the number of violations assigned by a faithfulness constraint $F$ to two candidates $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ and $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$ is never smaller than the number of violations assigned to their composition candidate $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}})$. I now show that this FTI provides an interpretation of (2) and (3).

(4)



(5) $F\big(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}}\big) \leq F\big(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}}\big) + F\big(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}}\big)$

■ Let me start with the FII$^{\mathrm{HG}}$ in (3). The FTI (5) and the antecedent of the FII$^{\mathrm{HG}}$ (3) obviously guarantee the consequent, so that (5) entails (3). The reverse entailment holds as well. In fact, assume that $F$ satisfies the FII$^{\mathrm{HG}}$ (3). Choose $\xi = F(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$, so that the antecedent of the implication (3) trivially holds. The consequent then holds as well and it is identical to the FTI (5). This straightforward reasoning yields the **second contribution** of this paper: for any faithfulness constraint $F$, (5) and (3) are equivalent. The technical condition for HG idempotence provided by the FII$^{\mathrm{HG}}$ (3) thus admits the following intuitive interpretation: it simply requires faithfulness constraints to measure phonological distance in a sensible way, namely in compliance with the triangular inequality.

■ What about OT idempotence? The FTI (5) obviously entails the FII$^{\text{OT}}$ (2). The reverse fails in the general case. Additional constraint assumptions are needed to bring the two conditions closer. McCarthy (2003) conjectures that phonological constraints are all *categorical*. Intuitively, this means that they assign at most one violation per *locus* of violation. McCarthy provides an explicit formalization of this intuition for markedness constraints (see his scheme (1) on p. 77). His treatment of faithfulness constraints is not as explicit: he discusses individual faithfulness constraints but does not provide a general scheme. The **third contribution** of this paper is an explicit theory of faithfulness categoricity. To illustrate the gist, consider $\textsc{Ident}_{\text{voice}}$. It can be defined in two steps. First, it is defined for candidates whose CR consists of a single corresponding pair $(\mathsf{a},\mathsf{b})$, as in (6a). Then, it is extended to arbitrary candidates by summing over all pairs in the CR, as in (6b).

(6) a. $\textsc{Ident}_{\text{vce}}\big(\mathbf{a},\mathbf{b},(\mathsf{a},\mathsf{b})\big)=\begin{cases}1 \text{ if } \mathsf{a} \text{ and } \mathsf{b} \text{ differ in voicing}\\ 0 \text{ otherwise}\end{cases}$

b. $\textsc{Ident}_{\text{vce}}\big(\mathbf{a},\,\mathbf{b},\,\rho_{\mathbf{a},\mathbf{b}}\big) = \sum_{(\mathsf{a},\mathsf{b})\in\rho_{\mathbf{a},\mathbf{b}}} \textsc{Ident}_{\text{vce}}\big(\mathbf{a},\,\mathbf{b},\,(\mathsf{a},\mathsf{b})\big)$

Other constraints such as $\textsc{Linearity}$ are defined analogously, but for the fact that the basic case concerns candidates whose CR consists of only two (not one) pairs so that the extension to a general candidate involves a sum over pairs of pairs of corresponding segments. Generalizing, a faithfulness constraint is called *CR-additive* of order $\ell$ provided it satisfies (7), where the sum is over any $\ell$ corresponding pairs. It is called *CR-categorical* provided it assigns either 0 or 1 violations to candidates whose CR consists of $\ell$ pairs, like those on the right-hand side of (7).

(7) $F\big(\mathbf{a},\,\mathbf{b},\,\rho_{\mathbf{a},\mathbf{b}}\big) = \sum_{(\mathsf{a}_1,\mathsf{b}_1),\ldots,(\mathsf{a}_\ell,\mathsf{b}_\ell)\in\rho_{\mathbf{a},\mathbf{b}}} F\big(\mathbf{a},\,\mathbf{b},\,\{(\mathsf{a}_1,\mathsf{b}_1),\ldots,(\mathsf{a}_\ell,\mathsf{b}_\ell)\}\big)$

CR-additivity entails *CR-monotonicity*: if the CR of a candidate is extended with additional corresponding pairs, the number of violations cannot but increase because the number of addenda on the right-hand side of (7) will increase. Since candidates are triplets of a UR, a SR, and a CR, besides CR-additivity/-monotonicity, *UR-* and *SR-additivity/monotonicity* can be defined analogously, roughly by summing over subsequences of length $\ell$ of the UR or SR string of segments. I go through the faithfulness constraints which have been proposed in the OT literature and show that they satisfy the following strengthened version of McCarthy's (2003) generalization: *they are all CR-categorical; or UR-categorical and SR-monotone; or SR-categorical and UR-monotone.*

■ This strengthened generalization motivates the equivalence (8) between the FII$^{\text{OT}}$ (2) and the FTI (5), which is the **fourth contribution** of this paper. Thus, also the condition for OT idempotence provided by the FII$^{\text{OT}}$ (2) admits the following intuitive interpretation: it requires the faithfulness constraints to measure phonological distances in compliance with the triangular inequality.

(8) **Thm.** Suppose that a faithfulness constraint $F$ is CR-categorical; or UR-categorical and SR-monotone; or SR-categorical and UR-monotone. Assume the correspondence relations in the candidate set are all one-to-one. $F$ satisfies the FTI (5) if and only if it satisfies the FII$^{\text{OT}}$ (2).

Here is an informal sketch of the proof. The FTI entails the FII$^{\text{OT}}$ in the general case. To prove the reverse implication, let me start with *binary* constraints, which assign 0 or 1 violations to any candidate. If the antecedent of the FII$^{\text{OT}}$ is true, its consequent is equivalent to the FTI. If the antecedent is false, the right-hand side of the FTI is at least 1 and the left-hand side can be at most 1 by binarity. Next, consider a (possibly non-binary) faithfulness constraint $F$ which is additive and categorical. Additivity means that the number of violations $F$ assigns to a candidate is the sum of the numbers of violations it assigns to the sub-candidates of order $\ell$. Categoricity means that $F$ assigns to these sub-candidates either 0 or 1 violations. In other words, $F$ is binary when restricted to the sub-candidates. The FII$^{\text{OT}}$ thus entails the FTI when restricted to the sub-candidates. By summing over all sub-candidates through additivity, the FTI for the original candidate follows. The UR/SR-monotonicity assumption in (8) is a technical condition needed for the proof machinery.

■ In conclusion, although the condition for HG idempotence provided by the FII$^{\text{HG}}$ (3) is stronger than the condition for OT idempotence provided by the FII$^{\text{OT}}$ (2) in the general case, the two conditions collapse (because both are equivalent to the FTI) for categorical/monotone faithfulness constraints which have been advocated in the literature. HG idempotence thus does not require additional constraint conditions. This equivalence in turn entails that the characterization of the faithfulness constraints which satisfy the FII$^{\text{OT}}$ in Magri (2015) yields an analogous characterization of the faithfulness constraints which satisfy the FII$^{\text{HG}}$, completing the theory of HG idempotence.