

# Stochastic constraint-based grammars for Hausa verse and song

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## 1. Singing is complicated; it comes in layers ...

- Three layers in Hausa (Chadic, W.Africa):
  - Words are arranged as poetry, to fit a **meter**.
  - The poetry is *re*-arranged in a **musical rhythm**, related to but not identical with the meter.
  - The sung rhythm is **realized in time**, in ways that are orderly but more than just mechanical reflections of musical rhythm.

## 2. Interest for phonologists here

- Extensive **variation** (even within a single poem), necessitating a formal model that can treat it.
- A new pattern of **harmonic bounding** — the implicational, maxent version.
- A bit of the renascent research program of **generative phonetics** — grammars of sound that go all the way to the physical surface.

## HAUSA POETRY AND SONG

## 3. Source of my knowledge

- The publications, data collection, and vast personal knowledge of my research collaborator, Prof. **Russell Schuh** of UCLA.

## 4. Hausa poetry is based on light and heavy syllables

- A syllable is **light** if it ends in a short vowel.    Symbol: ∪    “breve”    [ta]
- Else it is **heavy**.    Symbol: –    “macron”    [tan, taa]

## 5. Hausa meters

- A characteristic pattern of heavies and lights, defining a form of verse.
- For instance, here is the “catalectic mutadarik” meter, often used in Hausa:

$$\left\{ \begin{array}{c} \cup \cup \\ - \end{array} \right\} - \left\{ \begin{array}{c} \cup \cup \\ - \end{array} \right\} - \left\{ \begin{array}{c} \cup \cup \\ - \end{array} \right\} - \cup -$$

## 6. The importance of moras for Hausa verse

- Heavy syllables assumed to have two moras, lights one.
- We'll assume some sort of traditional moraic representation (e.g. Hyman 1985, Hayes 1989).
- Mutadarik above is a simple case of bimoraic equivalence; free substitution of bimoraic sequences.

## 7. There are quite a few meters in Hausa

- See Schuh's work in References; here we cover just one.

### THE RAJAZ METER

## 8. Basics of rajaz [ˈɪɑdʒɑz] structure

- Unit of composition is a **stanza** containing **five lines**.
  - Line 5 is special — see below.
- A **line** is composed of two<sup>1</sup> **metra** (sg. **metron**)
- A **metron** is normally composed of **six moras**.

## 9. The taxonomy of metron types

- There are **five major types** of metron, of which one hardly ever occurs initially, one hardly ever occurs finally.
- Combined counts of an 11 poem, 2476 line corpus:

Type	As Metron 1		As Metron 2		Both metra together	
	count	fraction	count	fraction	count	fraction
◡ – ◡ –	1146	0.463	521	0.210	1667	0.337
– – – –	173	0.070	864	0.349	1037	0.209
– ◡ ◡ –	336	0.136	494	0.200	830	0.168
– – ◡ –	688	0.278	<sup>47</sup>	<sup>0.019</sup>	735	0.148
◡ ◡ – –	<sub>25</sub>	<sub>0.010</sub>	487	0.197	512	0.103

## 10. A rajaz stanza that has all five metron types

- Stanza 32 of “Tutocin Shehu” (“The Banners of the Sheikh”) by Mu’azu Had’aja (1955)

◡ – ◡ – / – ◡ ◡ –      ◡ –  
 Wà.kíi.là    náa    mân.cè    wá.níi  
 Maybe    I-PERF forget    somebody  
 ◡            –            ◡ – / ◡      ◡ – –  
 Káa            sán    há.líi    dà    tù.nàa.níi  
 you-PERF know    manner    with memory

<sup>1</sup> Three, in trimeter rajaz.



## 14. Maxent metrics

- Sources:
  - **Maxent grammars:** Smolensky (1986), Goldwater and Johnson (2003); Hayes and Wilson (2008)
  - **As applied to metrics:** Hayes, Wilson and Shisko (2012)

## 15. Background on the maxent grammars to be developed here

- Maxent is one type of **Harmonic Grammar** (Smolensky 1986; Smolensky et al. 1992)
- It uses a variant of the **GEN + EVAL** architecture, as in Optimality Theory
- EVAL contains **constraints**, assigns **probabilities** to the candidates in GEN:
  - (very close to) **zero** for ungrammatical candidates
  - **positive probability** to acceptable candidates
  - The grammar **matches frequency** among multiple acceptable candidates.
- N.B.: no inputs or outputs, just GEN
- EVAL works like this:
  - Constraints are not ranked but **weighted**.
  - A mathematical formula ((24) below) translates violations and weights into predicted probabilities.
- How things work out in practice:
  - High-weighted constraints essentially **rule structures out**.
  - Weaker constraints determine **preferred** structures; they **match frequency** among the existing forms.

## 16. Our GEN

- In principle, all strings of  $\cup$  and  $-$  (infinite)
  - In practice: it is safe to use a finite set of 64 candidates for each metron.<sup>2</sup>
- We treat initial and final metra of the line separately, each with its own GEN.
  - This is safe: their patterning is statistically independent.
- We provide a separate GEN for stanza-final lines, which behave differently from non-final.
- Thus GEN = four lists of 64 candidates, each marked for a separate combination of 1st/2nd mora, stanza-final/non-final.

## 17. Constraints that (with the grid) enforce hexamoraicity

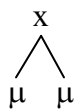
\*STRETCH: For every grid column greater than one to which a mora is associated, assess a penalty.

\*SQUEEZE: For every mora greater than one sharing a grid column, assess a penalty.

*Violation of \*STRETCH*



*Violation of \*SQUEEZE*

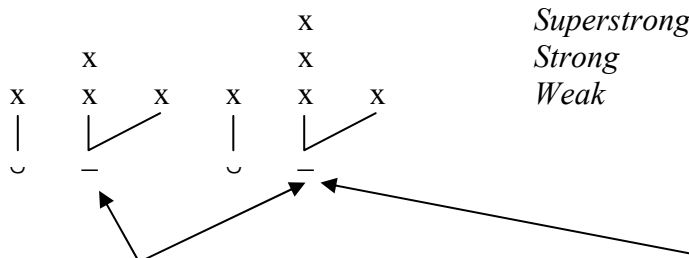


<sup>2</sup> We follow the method of Daland (in press), who establishes the conditions under which an infinite candidate set will not wreck a maxent analysis.

**18. Constraints that relate weight to grid column height (cf. (13))**

- These are **prominence alignment** constraints (Prince and Smolensky 1993)
  - similar to the well-known WEIGHT TO STRESS,
- They relate **weight** to **grid column height**.
  - a. STRONG IS LONG                      Assess a penalty for any Strong (or stronger) grid column that does not initiate a heavy syllable.
  - b. SUPERSTRONG IS LONG              Assess a penalty for any Superstrong grid column that does not initiate a heavy syllable.
  - c. LONG IS STRONG                      Assess a penalty for any heavy syllable that is not initiated in a Strong (or stronger) grid column.

**19. Effects of the constraints in (18)**



*Superstrong*  
*Strong*  
*Weak*

- STRONG IS LONG, a weaker constraint, enforces iambic preference.
- SUPERSTRONG IS LONG, a powerful constraint, enforces obligatory final heavy.

• In a minute we'll see the role played by LONG IS STRONG, which is also weak.

**20. Full analysis I: The really strong constraints**

- \*STRETCH, \*SQUEEZE, SUPERSTRONG IS LONG plus
- \*THREE LIGHTS IN A ROW (true in all Hausa meters, also active in Ancient Greek)
- \*DON'T START LINE WITH ~ ~ (a mystery from the viewpoint of theory, but certainly valid)

- Together, these limit common metra to:
  - First metron:     ~ ~ ~ -                      - - ~ -                      - ~ ~ -                      - - -
  - Second metron:     ~ ~ ~ -                      ~ ~ ~ -                      - ~ ~ -                      - - -

These are the only candidates in GEN that obey all five constraints.

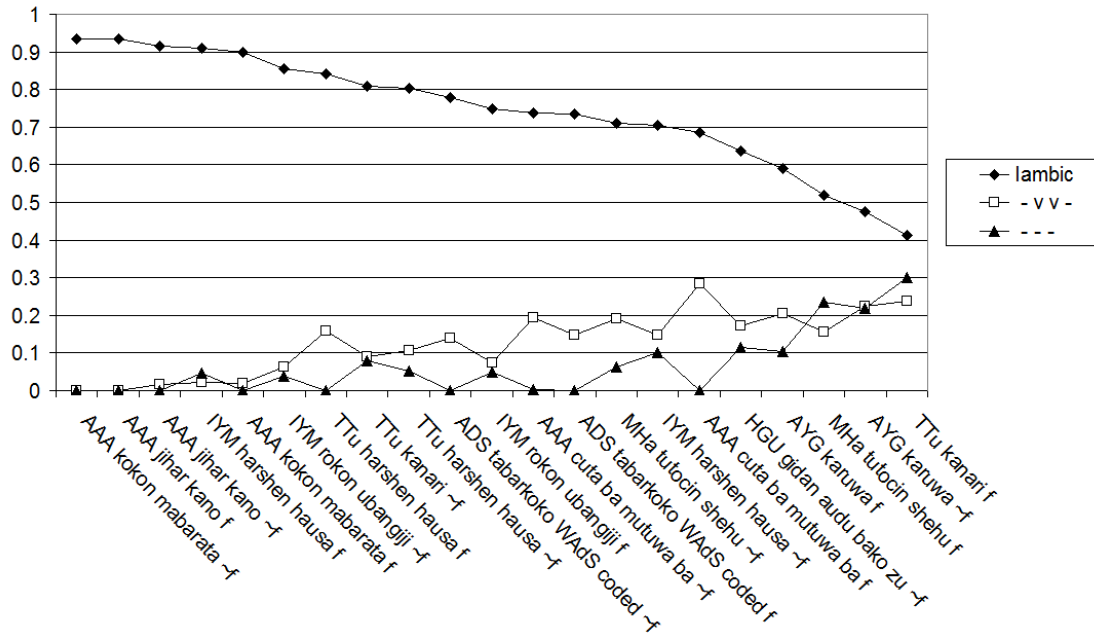
- Rest of the analysis: dealing with the **relative frequencies** of the five types.

**21. Metron 1: A key pattern**

- **First metra** vary by an orderly pattern:
  - Frequency of **iambic** ({[~ ~ ~ -],[ - ~ ~ -]}) > Freq. of [ - ~ ~ - ] > Freq. of [ - - - ].
  - True (with tiny exceptions) for every poem, both stanza-final and stanza-non-final.

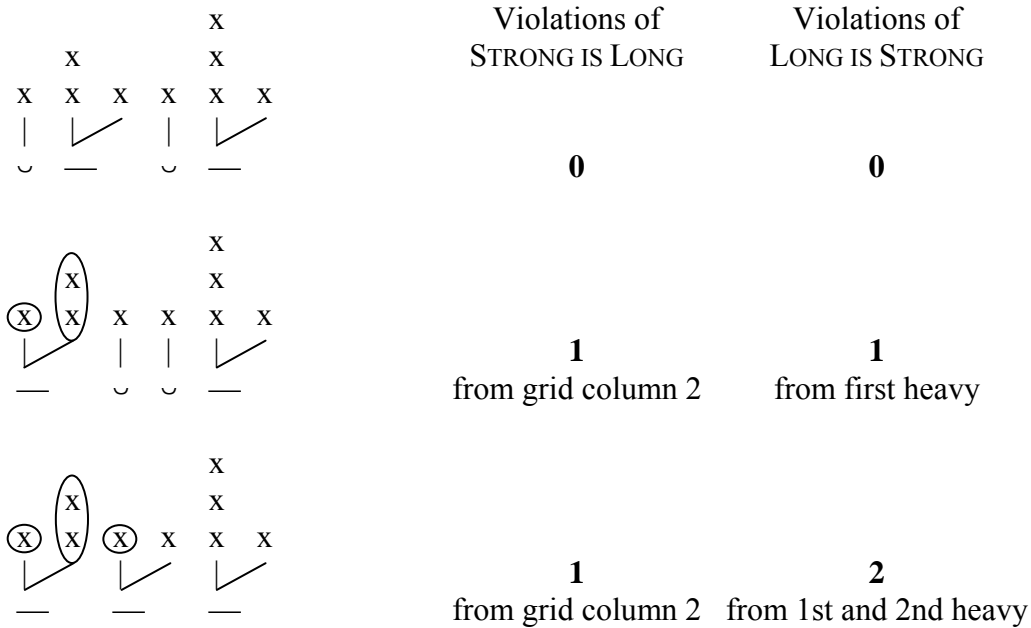
22. Graph: iambic > [- ∪ ∪ -] > [- - -] in the first metron

- f = final lines of stanzas, ~f = nonfinal lines



23. Can we derive this frequency relation from basic principles?

- Look at the violations of the constraints that discriminate these candidates:



- This is **harmonic bounding** (subsets of constraint violations), matching frequency pattern.

## 24. Capturing the generalization (22) with the math of maxent

- Here is the formula by which maxent derives predicted **probabilities** from the **violations** of candidates and the **weights**:

$$p(\omega) = \frac{1}{Z} e^{-\sum_i \lambda_i \chi_i(\omega)}, \text{ where } Z = \sum_j e^{-\sum_i \lambda_i \chi_i(\omega_j)}$$

$p(\omega)$  predicted probability of metron type  $\omega$   
 $\sum_i$  summation across all constraints  
 $\lambda_i$  weight of the  $i$ th constraint  
 $\chi_i(\omega)$  the number of times  $\omega$  violates the  $i$ th constraint  
 $\sum_j$  denotes summation across all possible metra

## 25. Deriving the implicational hierarchy just mentioned

- From the formula just given, it is easy to show that:
  - if Candidate A **harmonically bounds** Candidate B, then A necessarily gets **higher or equal probability** as B.
  - This is just what we observe in the frequencies of (22).
  - So poets can vary the weights of STRONG IS LONG and LONG IS STRONG as a part of their metrical style; but given harmonic bounding, they *must* reflect the *relative* frequency pattern.

## 26. Metron 2: much more is arbitrary

- Different poets favor different types for line-final metra
- ... and they often like to use a different pattern for the final metron of the final line of the stanza.
- We can attribute *some* of this variation to patterns of singing (below), and the presence of refrains, but we think it is primarily stylistic.
- We therefore include a number of rather arbitrary constraints that govern possible line-final and stanza-final metra.

## 27. Analyzing the poems in full: procedure

- We use the core constraints already seen — enforcing adherence to the rhythmic template
- Plus constraints of
  - Requiring particular line endings (we invoke “quantitative clausulae”, found in Slavic and Greek/Latin prose)
- Using software, we set the weights separately for every poem to match the data.<sup>3</sup>
- The “core constraints” virtually always get big weights.

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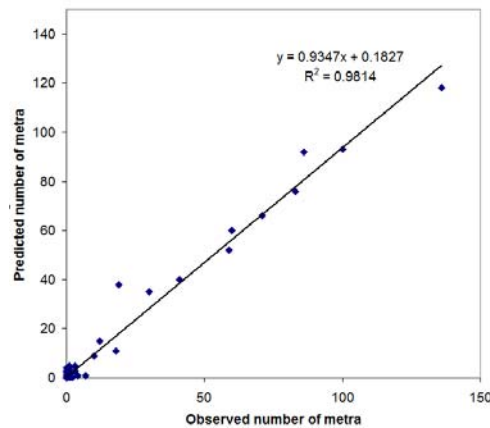
<sup>3</sup> Useful software: the “Solver” plug-in of Excel, or the Maxent Grammar Tool ([www.linguistics.ucla.edu/people/hayes/MaxentGrammarTool/](http://www.linguistics.ucla.edu/people/hayes/MaxentGrammarTool/))

## 28. The weights for one poem, “Tutocin Shehu”

*SQUEEZE	4.2	* $\cup \cup \cup$	1.4
*STRETCH	3.6	--- CLAUSULA (LINE-FINAL)	0.8
SUPERSTRONG IS LONG	3.3	DON’T EMPLOY “INITIAL HEAVY AS LIGHT” CONVENTION	0.7
STRONG IS LONG — last metron of stanza	2.9	LONG IS STRONG	0.5
DON’T START LINE WITH $\cup \cup$	2.4	STRONG IS LONG	0.4

## 29. Model fit

- Not bad; **scattergram** of predicted and observed frequencies is given at right.
- 256 data points: one GEN of 64 types for each of the four combinations of Metron 1/Metron 2, Stanza-final/Stanza-nonfinal.
- Most data points are near the origin.
- Other poems have very different metron patterns but similar accuracy of the grammar’s predictions.<sup>4</sup>



## SINGING THE RAJAZ

## 30. Data

- Schuh’s collection has recordings of five artists singing poetry in rajaz.
- **Not one** sings it “straight”, with the grid given above in (13).
- Every time, we see **remapping** of the basic pattern of the meter onto a new pattern for song.
  - This is also common in English and other European languages.
- Let’s look at two such remappings.

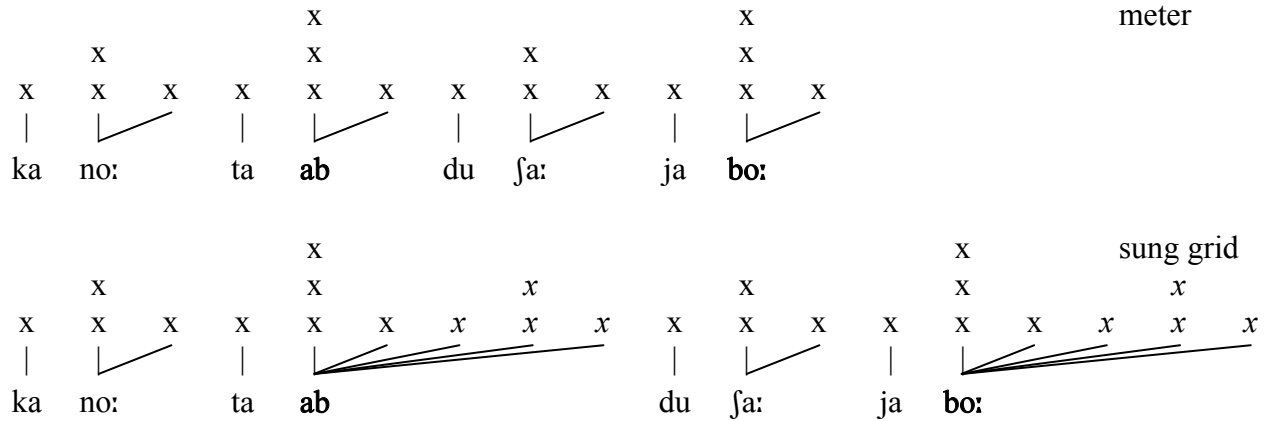
## 31. Remapping in “Tutocin Shehu” as sung by Abubakar Ladan

- The rhythm is what in Western music we would call 9/8 time.
- Grid is “augmented” with spliced-in material.

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There are three other constraints active for poems other than “Tutocin Shehu”: --- CLAUSULA (LINE-FINAL),  $\cup \cup$  - CLAUSULA (LINE-FINAL), and --- CLAUSULA, STANZA FINAL.



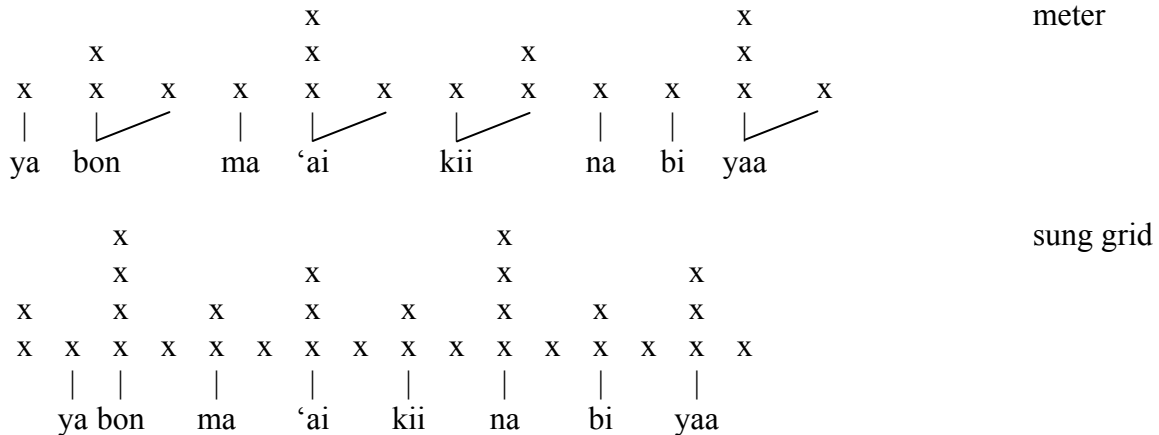


Kánò tá Ábdù sháa yàbóo (line 1a)  
 Kano of Abdu receive praise  
 ‘Kano of Abdu be praised’

[ listen ] Please ignore the quasi-disyllabic character of [ab].

**32. Kokon mabarata,<sup>5</sup> by Ak’ilu Aliyu, sung by the poet**

- Binary rhythm with short upbeat.
- Striking (but not total) disregard for syllable quantity — e.g. heavy syllables tend to fall in the stronger positions.



**33. Analysis of “remapping”**

- We have more questions than answers here!
- One very simple theory:
  - Poetry gets written as rajaz.
  - Meter is *thrown away*, leaving pure text.
  - Pure text is then sung to a new grid — no “grid-to-grid” connection<sup>6</sup>

<sup>5</sup> “Alms Seekers’ Bowl”

- Yet — remapping is also found in improvised poetry (Schuh, forthcoming)
  - Can a poet *simultaneously* respect the requirements of two different grids?
  - These issues — “how many levels, how do they interact” — are cogently raised by Kiparsky (2006) for English folk verse, and explored in depth for Tashilhiyt Berber song by Dell and Elmedlaoui (2008).

## MODELING THE PHONETICS OF RAJAZ SINGING

### 34. Generative phonetics

- Any theory that formally models the productive capacity of the native speaker to form novel phonetic representations and realize them physically in sound.

### 35. We are witnessing a revival of generative phonetics.<sup>7</sup>

- **Constraint-based models** (with GEN and EVAL) can directly express the functionally-sensible, conflicting teleologies we see in phonetics.
- **Harmonic grammar** is easily extended to treat the quantitative data.
- **Effective learning models** let us tune the phonetic grammars closely to language data — perhaps as children do when they acquire the phonetic pattern of their native language.
- Some standout work: Boersma (1998 et seq.), Flemming (2001), Flemming and Cho (2015), Windmann et al. (2015)

### 36. A fundamental principle of phonetic realization: *quantitative compromise between conflicting targets*

- **Example:** adding syllables to a word lengthens the word, but by less than the duration of the syllables added — since the original word shortens.
- Original reference is Lehiste (1972); see Fletcher (2012) for literature review.

<i>sum</i>	<i>summon</i>	<i>summoner</i>
longest [sʌm]	medium [sʌm]	shortest [sʌm]
546 ms.	348 ms.	273 ms.
shortest word	medium word	longest word
[sʌm]	[sʌmən]	[sʌmənər]
546 ms.	570 ms.	580 msec.

- Duration of [sʌm] is a compromise between its own inherent duration, and desire for words not to be too long.
- Phonetics of duration: stuffing **compressible sponges** into **stretchable sacks**.

<sup>6</sup> There are very clear examples of this; e.g. in the famous “Queen of the Night aria”, Mozart utterly obliterates the iambic pentameter rhythm provided by his librettist Emanuel Schikaneder.

<sup>7</sup> Two outstanding works of rule-based generative phonetics: Allen et al. (1987), Beckman and Pierrehumbert (1988)

### 37. Implementing compromise in maxent grammars: Flemming (2001)

- His “maxent” is actually a non-stochastic version, generating one single output.
- Let linguistic categories each have a quantitative **target value**.
- Constraints penalize deviations from the value.
  - Actually: squared deviations, since otherwise you get many tied winners.

### 38. What Flemming accomplished with this model

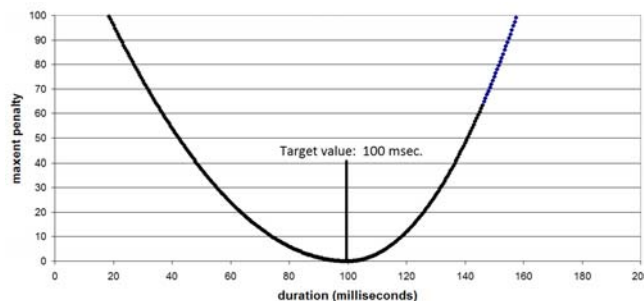
- Derived — as a theorem — the most famous of all compromise effects in phonetics: the **locus** effect in stop consonants (Sussman et al. 1991)
- Formant frequency at consonant release is a compromise between vowel target and consonant target.

### 39. “Sponges” and “sacks” for phonetic duration grammars

- \*SQUEEZE: violations = square of [candidate value – target value] (in milliseconds)
- \*STRETCH: violations = square of [target value – candidate value] (in milliseconds)
- We use both constraints for all targets, hence everything is both a sponge and a sack.

### 40. A schematic chart relating maxent penalty to deviation from target

- Weight of \*SQUEEZE is 10; \*STRETCH is 20.
- Since penalty is based on squared distance, we get two half-parabolas, steeper for \*STRETCH.



### 41. Moving toward the rajaz analysis

- We set up **duration targets** for each of these categories:
  - syllable
  - mora
  - grid column
  - metron
- Each target is regulated by a \*STRETCH and a \*SQUEEZE constraint.
- We “**maxentify**” Flemming:
  - utilizing the maxent **learning procedure**
  - System outputs not a single value but a **probability distribution**.

#### 42. The set of candidates (GEN)

- A three-syllable metron should have a GEN that specifies all combinations of three nonnegative real numbers, one for each syllable.
- But this is uncountably infinite ...
- In practice, use **Boersma's idealization** (Boersma 1998)
  - all values on a fine-spaced grid (every 10 msec.) that are ...
  - within a particular range covering all actual values (80 - 490 msec.)

#### 43. The data we model

- Syllable durations of 100 lines of Tutocin Shehu
- Last syllable of metron hard to measure (often prepausal; “fades out”) and we omitted these syllables from the modeling
  - Hence our “metron” target is really a “hemimetron” target.
  - Total syllables modeled = 562

#### 44. Learning the grammar from data

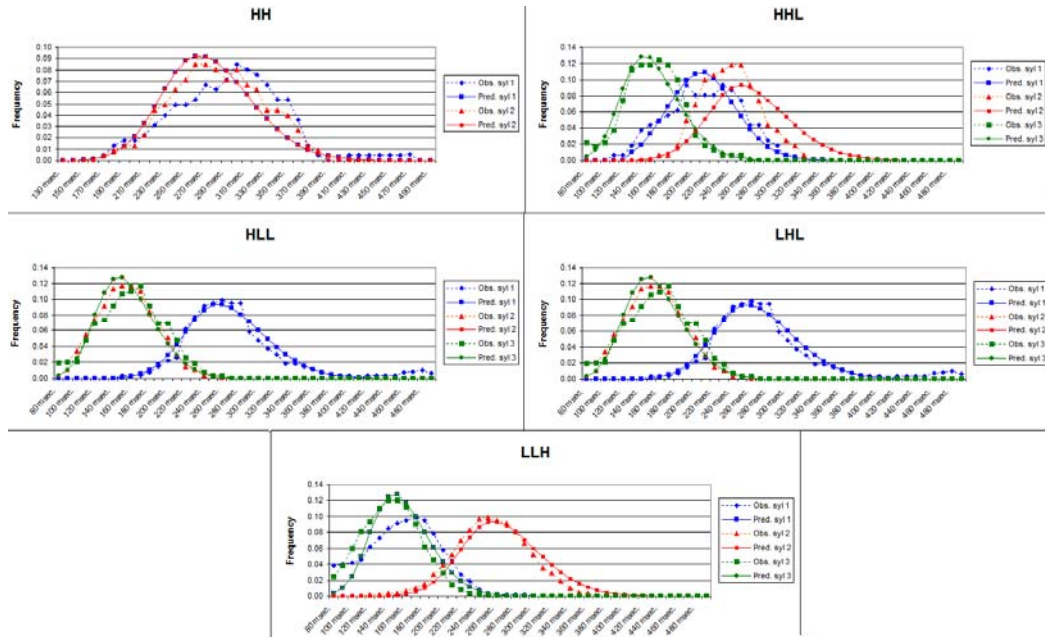
- To get the right weights, we use a machine search, following the method laid out in Flemming and Cho (2015).
- This is hard (local maxima of search space); please consider this work-in-progress.

#### 45. The 12 parameters of our phonetic grammar

- Durational targets for four categories:
  - Mora: 110 msec.
  - Syllable: 187 msec.
  - Grid column: 129 msec.
  - Hemimetron: 573 msec.
- Weights for four \*STRETCH constraints based on these targets:
  - \*STRETCH MORA: 3.17
  - \*STRETCH SYLLABLE: 2.07
  - \*STRETCH GRID COLUMN: 4.87
  - \*STRETCH HEMIMETRON: 0.88
- Weights for four \*SQUEEZE constraints based on these targets:
  - \*SQUEEZE MORA: 1.96
  - \*SQUEEZE SYLLABLE: 0.03
  - \*SQUEEZE GRID COLUMN: 0.63
  - \*SQUEEZE HEMIMETRON: 0 (hence no effect)

**46. This ends up working fairly well**

- General fit is not bad, with predicted *distributions* (solid) matching *distributions* (dotted).

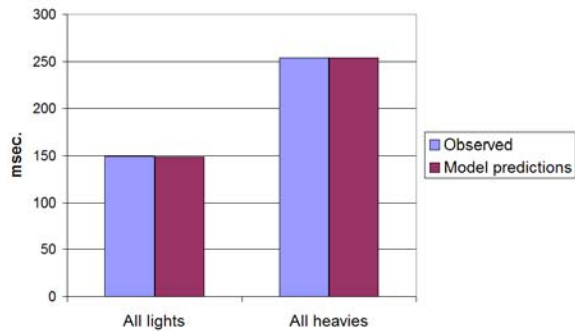


**47. Reasoning through this qualitatively**

- We will look at four compromises that get made and how the model captures them.

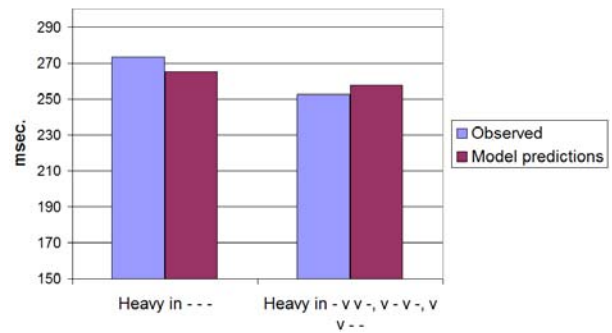
**48. Heavy syllables are not twice as long as two lights**

- Key idea: syllable target is lower than the doubled value of mora target. Maxent compromises.



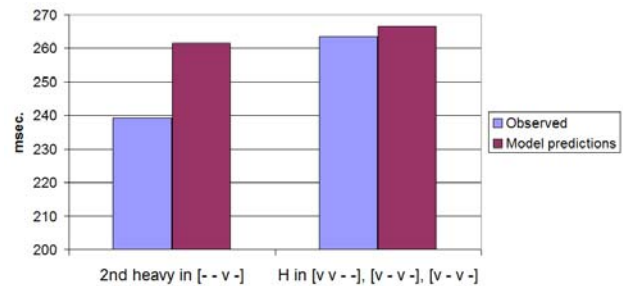
#### 49. Heavies in [---] are longer than heavies in [v v ---], [v - v -], [- v v -]

- Key idea: The moras of light syllables are “fat” moras. Syllables of [---] do not have to share the metron with “fat” moras and have more room.
- Model predicts a difference in the right direction; a better model would predict a bigger difference.



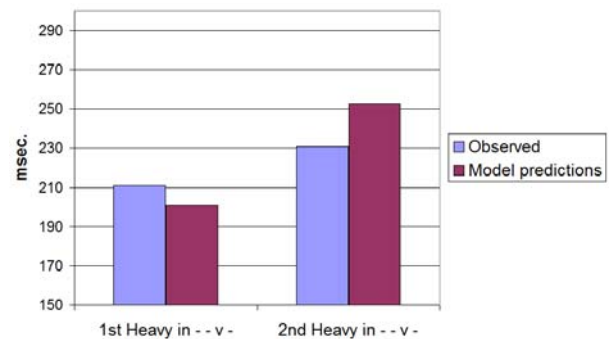
#### 50. Heptamoraic [- - v -] should have the shortest syllables of all

- Key idea: an extra mora must share the metron — one more “sponge in the sack”
- Model predicts a difference in the right direction; a better model would predict a bigger difference.



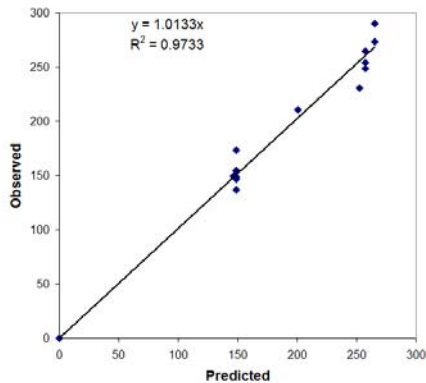
#### 51. Of the first two heavies in [- - v -], the first should be the shortest

- First heavy fills a single grid slot, due to the “count initial heavy as light” convention (11).
- Analysis: compromise between syllable and mora targets vs. grid column target.
- Model predicts the difference, but exaggerates it.



## 52. Overall model accuracy

- Scattergram of predicted vs. observed frequency for all 14 “types” of syllable (cases that the grammar treats distinctly).



## 53. Pondering the result

- Maxent **naturally derives the compromises** observed in phonetic patterning
- Here, the compromise involves **all** levels discussed. Initial syllable of heptamoraic metron is:
  - **longer** because it is **phonologically heavy**
  - **shorter** because the **metrics** assigns it to one grid column
  - **also shorter** because the sung-grid metron must accommodate seven moras
- The phonetic system is clearly **adaptable**, since the phonetic targets of the sung rhythm are superimposed on the phonological targets.

## 54. Overall conclusions

- We think we are doing pretty well modeling the metron frequencies of the 11 poems, using maxent and a mixture of natural and stylistic constraints.
- The remapping from verse rhythm to sung rhythm is at present a puzzle for theory — but at least we have some clear cases that could be used to help figure it out.
- Durational modeling is reasonably accurate and uses what seem to be sensible constraints.

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